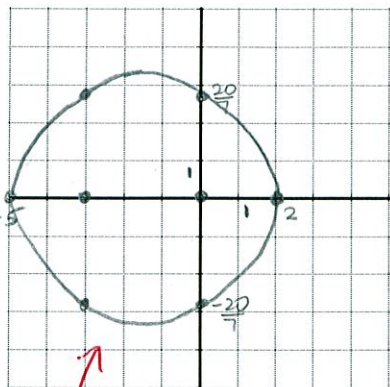


Consider the graph of the polar equation  $r = \frac{20}{7 + 3\cos\theta} = \frac{\frac{20}{7}}{1 + \frac{3}{7}\cos\theta}$

SCORE: \_\_\_\_ / 9 PTS



[a] Fill in the blanks.

[i] The eccentricity is  $\frac{3}{7}$  ①

[ii] The shape of the graph is a/an ELLIPSE ①

[iii] The equation of the directrix is  $x = \frac{20}{3}$  ①

[iv] Find the rectangular coordinates of the

x - intercept(s)

$(-5, 0), (2, 0)$  ①

y - intercept(s)

$(0, \frac{20}{7}), (0, -\frac{20}{7})$  ①

focus/foci  $2 + 5 = -3$

$(-3, 0), (2, 0)$  ①

endpoints of the latus rectum/latera recta

$(-3, \pm \frac{20}{7}), (2, \pm \frac{20}{7})$  ①

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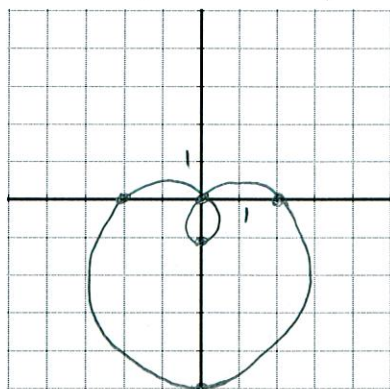
$\theta$	$r$
0	$\frac{20}{10} = 2$
$\frac{\pi}{2}$	$\frac{20}{7} = 2\frac{6}{7}$
$\pi$	$\frac{20}{4} = 5$
$\frac{3\pi}{2}$	$\frac{20}{7} = 2\frac{6}{7}$

[b] Sketch the graph on the grid provided above. You must provide a scale for the axes & plot all points from part [a][iv] above.

Consider the graph of the polar equation  $r = 2 - 3\sin\theta$ .

$0 < |\frac{2}{-3}| < 1$

SCORE: \_\_\_\_ / 6 PTS



[a] Fill in the blanks.

[i] The shape of the graph is a/an LIMACON WITH LOOP ①

[ii] The graph DOES ① pass through the pole.  
does / does not

[iii] Find the rectangular coordinates of the

x - intercept(s)

$(0, 0), (2, 0), (-2, 0)$  ①

y - intercept(s)

$(0, 0), (0, -1), (0, -5)$  ①

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$\theta$	$r$
0	2
$\frac{\pi}{2}$	-1
$\pi$	2
$\frac{3\pi}{2}$	5

[b] Sketch the graph on the grid provided above. You must provide a scale for the axes & plot all points from part [a][iii] above.

Consider the graph of the polar equation  $r = 2 - 2\sin 2\theta$ .

SCORE: \_\_\_\_ / 15 PTS

**NOTE:  $(r, \pi - \theta)$  and  $(-r, \pi - \theta)$  tests do NOT show that the graph is symmetric**

- [a] Using the information above, and the tests and shortcuts shown in lecture, test if the graph is symmetric over the pole, the polar axis, and/or  $\theta = \frac{\pi}{2}$ . State your conclusions in the table. **NOTE: Run as FEW tests as needed to prove your answers are correct.**

$\theta = \frac{\pi}{2}$ :  $-r = 2 - 2\sin 2(-\theta)$

$-r = 2 + 2\sin 2\theta$

①  $r = -2 - 2\sin 2\theta$  x

POLAR AXIS:  $r = 2 - 2\sin 2(-\theta)$

①  $r = 2 + 2\sin 2\theta$  x

POLE:  $r = 2 - 2\sin 2(\pi + \theta)$

$r = 2 - 2\sin(2\pi + 2\theta)$

$r = 2 - 2[\sin 2\pi \cos 2\theta + \cos 2\pi \sin 2\theta]$

①  $r = 2 - 2\sin 2\theta$  ✓

★  
"NO"  
AND  
"NOT  
SYMMETRIC"  
ARE INCORRECT

Type of symmetry	Conclusion
Over the polar axis	NO CONCLUSION
Over $\theta = \frac{\pi}{2}$	NO CONCLUSION
Over the pole	SYMMETRIC

② IF ALL 3 CORRECT  
① IF 2 OF 3 CORRECT  
③ IF NONE OR ONLY 1 CORRECT

- [b] Based on the results of part [a], what is the minimum interval of the graph you need to plot first (before using reflections to draw the rest of the graph)?

$[0, \pi]$  OR  $[-\frac{\pi}{2}, \frac{\pi}{2}]$  ① EITHER ANSWER IS OK

- [c] Find the angles in the minimum interval in part [b] at which the graph goes through the pole.

$0 = 2 - 2\sin 2\theta$

①  $\sin 2\theta = 1$

IF USING  $[0, \pi]$ ,

$0 \leq 2\theta \leq 2\pi$

so  $2\theta = \frac{\pi}{2}$

①  $\theta = \frac{\pi}{4}$

IF USING  $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$-\pi \leq 2\theta \leq \pi$

so  $2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

- [d] Find the value of  $r$  for all common angles in the minimum interval in part [b].

$\theta$

0

①  $[2]$

$\frac{\pi}{6}$

①  $[2 - \sqrt{3} \approx 0.2]$

$\frac{\pi}{4}$

0

$\frac{\pi}{3}$

①  $[2 - \sqrt{3} \approx 0.2]$

$\frac{\pi}{2}$

①  $[2]$

$\theta$

$-\frac{\pi}{2}$  or  $\frac{2\pi}{3}$

①  $[2 + \sqrt{3} \approx 3.8]$

$-\frac{\pi}{4}$  or  $\frac{3\pi}{4}$

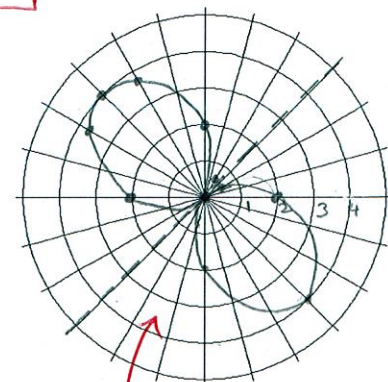
①  $[4]$

$-\frac{\pi}{6}$  or  $\frac{5\pi}{6}$

①  $[2 + \sqrt{3} \approx 3.8]$

$-\frac{\pi}{2}$  or  $\pi$

①  $[2]$



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BY ME

- [d] Sketch the graph on the grid provided below. **You must provide a scale for the polar axis & plot all points from part [c] above.**